

The effect of background rotation on fluid motions: a report on Euromech 245

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The 245th Euromech Colloquium, on the effect of background rotation on fluid motions, was held in Cambridge from 10–13 April 1989 with the two authors acting as chairmen. There were sixty-five participants with widely different backgrounds. The striking feature of the Colloquium was the recent advances on nonlinear processes. A wide range of nonlinear phenomena was presented and particular emphasis was on the formation and dynamics of coherent vortices. The similarities between the processes in rotating, curved and stratified flows, which lead to anisotropic motions and long-lived coherent structures alongside linear wave motions was a feature of many of the presentations. Fifty-one papers were presented, covering engineering, geophysical and astrophysical applications of rotating fluids. These papers are summarized in this report with the purpose of giving an up-to-date view of current research in rotating fluids.

1. Introduction

As far as we are aware, the Euromech meeting on rotating fluids was the first of this type to be held since the 1965 IUTAM Symposium (Bretherton, Carrier & Longuet-Higgins 1966). We therefore thought it important to cover as wide a range of topics as possible.

The topics discussed at the meeting reflect the present day research activity on rotating fluids; these were

- (i) barotropic shear flows,
- (ii) effects of rotation on turbulence,
- (iii) flow over obstacles,
- (iv) vortex flow and centrifuges,
- (v) convection in rotating fluids,
- (vi) geostrophic vortices,
- (vii) geostrophic adjustments and waves,
- (viii) baroclinic and barotropic instabilities.

Some of these subjects were discussed at the San Diego meeting, but what clearly emerged during the present meeting was a significant development in the work on nonlinear processes and turbulence. Elegant experimental work was described and numerical simulations of complex flows are now possible. Three topics (i, ii and vi) were introduced by review lectures because of their general interest and also because progress was particularly substantial. Two other topics would have deserved review

lectures, topics (iv) and (viii), and these were planned but had to be cancelled at the last moment. Compared with the IUTAM Symposium there is a relative absence of papers on wave motions at the present meeting.

One of the main objectives was to have an interdisciplinary meeting emphasizing geophysical and astrophysical as well as engineering applications. Many fundamental processes are of common interest and engineers can learn much from what has been done in geophysics and vice versa. In particular, the effects of rotation on shear flows and on turbulence and also vortex dynamics and stability are topics of common interest.

In this report the papers are grouped together basically in the same way as was adopted for the Colloquium. Some minor rearrangements are made in order to facilitate the discussion of related papers. Additional, up-to-date references are included where it was thought that these would add valuable information.

2. Barotropic shear flows

The survey lecture given by D. J. Tritton* concentrated on shear flows in which the shear vorticity and the system rotation vector are either parallel or antiparallel. Both wall-bounded flows and free turbulent shear flows were discussed. For nearly parallel flows the importance of rotation depends on the sign and value of the parameter

$$S = -2\Omega/(\partial U/\partial y),$$

where Ω is the angular velocity and $\partial U/\partial y$ the mean velocity gradient normal to the rotation axis. When $S < 0$ rotation is destabilizing and when $S > 0$ rotation is stabilizing. Maximum destabilization occurs for $S = -\frac{1}{2}$. For stronger rotation rates restabilization would again be expected. This is inferred from the Reynolds stress equation and also from the equivalent Richardson number argument, $Ri = S(S+1)$, introduced by Bradshaw (1969). The lowest value of Ri is $-\frac{1}{4}$ and is achieved when $S = -\frac{1}{2}$. The effect of stabilization and destabilization of rotation on turbulent channel flow has been demonstrated by the experiments of Johnston, Hallen & Lezius (1972), but there is, as yet, no experimental evidence for the possibility of restabilization when $S < -\frac{1}{2}$.

In free shear flows such as a shear layer or a wake, rotation can either enhance the two-dimensional vortex structure or destroy it, depending on whether the rotation vector is parallel or antiparallel, respectively, to the shear vorticity. In addition, in a stabilized shear layer experiments show (Tritton 1985) that vortex pairing seems to be inhibited. During the discussion as to why this should happen, it was speculated that in the experiments of Tritton, Ekman friction may be the cause of this effect.

Results of numerical simulations of turbulent channel flow with background rotation were presented by Morchoisne, Teissedre & Dang*. Calculations were made for a Reynolds number $U_c b/\nu = 1650$, where U_c is the centreline velocity and b the channel half-width. A semi-implicit scheme and an $18 \times 32 \times 65$ mesh was used. Results were reported for only one value of rotation rate, and the calculations concentrated on the change in wall shear stress on the stable and unstable sides. Good agreement with the experiment by Johnston *et al.* (1972) was found even though Reynolds numbers differed by one order of magnitude.

Alfredsson & Matsson* first reported results of a stability analysis of a rotating channel flow, the results of which can be found in Alfredsson & Persson (1989). The

* Papers presented at the Colloquium are indicated by an asterisk both in the text and in the list of references.

main part of the presentation was, however, concerned with the stability of curved and rotating channel flow, the rotation axis being parallel to the walls and normal to the flow. The linear stability analysis of this flow, in the case when centrifugal and Coriolis effects counteract each other, leads to a neutral stability curve which has two loops, with the unstable region being determined by the envelope to all curves of constant positive growth rate. Experiments performed by the same authors confirmed results of the stability analysis and, furthermore, indicated that the secondary instabilities on the longitudinal rolls, which are observed at Reynolds numbers well above the critical value for onset of the rolls in the non-rotating case, are completely suppressed by rotation.

Reynolds stress modelling can capture the physics of rotation effects on turbulent shear flows. This was demonstrated by Andersson & Nilsen* who used an algebraic Reynolds stress model to calculate the effect of rotation of an unidirectional shear flow, $U_1(x_2)$, with the rotation axis parallel to x_2 . The model gives good results for the rotationally stable as well as the unstable configuration. In particular, it reproduces the experimentally observed apparent saturation of the rotationally produced increase in Reynolds stress for sufficiently large values of $|S|$, corresponding to values of the overall rotation number $\bar{S} = 2\Omega b/u > 0.2$. The model also shows that the isotropizing effect of positive rotation is opposed by the presence of solid boundaries. The question of whether or not strong rotation ($S < -\frac{1}{2}$) would 'restabilize' the unstable side is still an open one, and it is doubtful that Reynolds stress models would be of any help. Hopfinger (1989) conjectured that restabilization could occur when the turbulent Rossby number falls below about 0.2.

Hunt & Hussain* presented new results for the integral of helicity

$$H = \int_V \mathbf{u} \cdot \boldsymbol{\omega} dV$$

of the flow *outside* a closed spherical fluid volume moving in a inviscid rotating ambient fluid. Explicit expressions for the helicity integral were obtained; surprisingly helicity is negative when the fluid volume moves in the direction of the rotation vector and positive when it moves opposite to it. The objective was to develop new physical concepts of helicity in turbulent rotating flows. It is generally believed that the large values of helicity are indicative of ordered structures in the turbulent flow.

The following three papers of the session considered shear flows driven by rotating boundaries but which were not subject to background rotation. Bühler* reported numerical and experimental results on the flow generated in a spherical gap between two concentric spheres, with the outer one being at rest and the inner one rotating. The novelty of his experiment was that the inner sphere consisted of two hemispheres which could be rotated independently. Results were presented for three cases: counter-rotating hemispheres, one rotating hemisphere and differential rotation between the two. In the counter-rotating situation the secondary flow in the meridional plane always gives rise to toroidal vortices positioned symmetrically with respect to the equatorial plane. Since in this plane the centrifugal force is cancelled owing to the counter-rotation, no critical Reynolds number is associated with the appearance of these toroidal vortices. When the Reynolds number is increased, the shear layer in the meridional plane becomes unstable and transitions to steady three-dimensional modes occur. These transitions have a hysteresis behaviour (Bühler 1989). In the case of differential rotation at supercritical Reynolds numbers the flow

states are not unique and depend on the rate of increase or decrease in rotation rate of one hemisphere.

In the above spherical gap flow the ratio of the gap width to sphere radius was small. For large gap width, interesting phenomena can also be observed in the polar regions as was shown by Bar-Yoseph, Solan & Roesner*. The meridional secondary flow leads to vorticity concentration in the polar region and for large Reynolds numbers

$$Re = \Omega \frac{R^2}{\nu} \geq 3000,$$

where R is the radius of the inner sphere, vortex breakdown (local recirculating flow in the vortex) is observed (Bar-Yoseph *et al.* 1987) in a way similar to that found in a cylindrical container with a rotating lid (see Spohn, Mory & Hopfinger*). Roesner showed experimentally that an axial eccentricity can enhance this process and that when the Reynolds number is increased above the critical value for breakdown period motions of the circulating region can be observed. These fascinating phenomena were visualized by an UV-light colouring technique using a saturated solution of photochromic spiropyrans in an organic liquid (see for instance Larsen & Roesner 1982).

Naimi, Devienne & Lebouché* reported results on the instability in Taylor-Couette flow of a yield-pseudoplastic fluid. It was found that the critical Taylor number was larger than the corresponding value for Newtonian fluids and that toroidal vortices first appeared at the ends of the annular gap. An imposed axial flow had, as expected, a stabilizing effect.

3. Effects of rotation on turbulence

The effect of background rotation on homogeneous three-dimensional turbulence was introduced by Cambon* in his review lecture on modelling of turbulence subjected to rotation. It was pointed out by Cambon that rotation has essentially two effects on turbulence: one is a reduction of the dissipation rate and the other is a strong departure from isotropy of the integral lengthscales, with the Reynolds stress tensor remaining nearly isotropic. The effects on the dissipation rate are accounted for (Bardina, Ferziger & Rogallo 1985) by a modification of the dissipation in a (k - ϵ) turbulence model. The mechanisms of anisotropization can be understood by using a two-point closure model which takes into account the dispersive and anisotropic characteristics of inertial waves (Cambon & Jacquin 1989). In the model of Cambon, linear and nonlinear effects were treated separately. From the understanding of the mechanisms gained from the two-point model, Cambon suggests a one-point closure model of the type $k, u_i^2, L_{ii,j}$ where $L_{ii,j}$ denotes the integral lengthscales in the various directions, or a $u_i u_j, \epsilon$ model as was used for rotating turbulent shear flows (see Andersson & Nilsen*)

Experiments which support the results of the model discussed by Cambon were presented by Jacquin*. Grid turbulence of Taylor microscale Reynolds number of about 40 was subjected to background rotation. Relatively low turbulent Rossby numbers

$$Ro = \frac{u_1}{2\Omega L_{11,1}} \geq 0.1$$

could be achieved in the experiments, where $L_{11,1}$ and u_1 are the longitudinal integral scale and the r.m.s. turbulent velocity parallel to the rotation axis, respectively. Two breakpoints were observed: one at $Ro \approx 1$ where the ratio $2L_{22,1}/L_{11,1}$ started to

deviate from the isotropic value of 1, to reach a value of about 2 at $Ro \approx 0.2$, and the other at $Ro \approx 0.2$ below which $L_{11,1}$ and u_1 increased weakly compared with the non-rotating values.

The combined effects of stable stratification and rotation on homogeneous turbulence were considered by Métais*, who extended the numerical code used by Métais & Herring (1989) for the simulation of stratified turbulence to include rotation with the rotation axis parallel to the direction of stratification. It was found that with rotation the vertical variability is reduced and that two-dimensionalization, expressed by the vertical vorticity energy, is enhanced by rotation.

The last paper in this session on rotation effects on turbulence and in particular on turbulent mixing was given by Fleury & Mory*. Turbulence without mean flow was generated by an oscillating grid and subjected to rotation in a way similar to Hopfinger, Browand & Gagne (1982). In contrast to the latter experiments the fluid column in Fleury & Mory's experiment was a two-layer stratified fluid. The density interface was positioned in the zone where the turbulent Rossby number Ro was in the range $0.2 \leq Ro \leq 1$ where the turbulence decay rate is little affected by rotation, and also in the region $Ro < 0.2$ where the turbulence structure is rotationally dominated. Results concerning the entrainment rate across the interface, expressed by the dimensionless entrainment rate u_e/u were reported for a range of Richardson (Ri) and Rossby (Ro) numbers. The data were correlated by the expression

$$\frac{u_e}{u} \approx 0.5Ro Ri^{-1}.$$

Frequency spectra of the local interface displacements showed significant modifications of the dynamics of the interface in the presence of rotation. It was also pointed out that inertial waves are capable of transmitting a large fraction of the turbulent kinetic energy across the interface.

4. Flows over obstacles

Chabert d'Hieres, Davies & Didelle* presented results of the effects of rotation on lift and drag coefficients for right circular cylinders in a rotating homogeneous fluid with the axis of the cylinder being aligned with the rotation axis. The experiments were done on the large turntable in Grenoble where it is possible to reach Reynolds numbers of 10^4 for Rossby numbers, based on cylinder diameter, as low as 0.1 and Ekman numbers of order 10^{-6} . Values of the lift and drag coefficients for a full and truncated cylinder were given and it was shown that the lift coefficient has a Ro^{-1} dependence, whereas the drag coefficient was found to be independent of Ro for values $Ro > 0.2$. Lift and drag coefficients had larger values for the truncated cylinder because, owing to the presence of a Taylor column, the effective cylinder length extends over nearly the whole fluid layer.

Flow separation conditions and the resulting wake structures in rotating stratified flow over a surface-mounted hill was investigated by Smeed*. The situation considered was for small-scale topography (in the geophysical sense) with Rossby number, based on hill diameter of order one, $0.2 < Ro < 4$. The flow is predominantly around the hill when the Froude number F , based on the hill height, is below a critical value of 0.5 and over the hill when $F > 0.5$ (Smeed 1990). Smeed covered the range of Froude numbers from below to above critical, $0.15 < F < \infty$, and was able to characterize five distinct wake flow regimes. For small hill heights a linear model is capable of calculating stresses on the hill surface and the flow outside the boundary layer that compare well with experiments.

In both the ocean and the atmosphere, vortex streets behind islands or hills are observed (see for instance Thomson 1977), and Etling* showed various examples of the occurrence of such vortex streets in the atmosphere. The typical scale of the shed vortices is 50 km in diameter where the islands are flat (2 km high) and small (40 km base) three-dimensional obstacles. A stable stratification is therefore necessary for two-dimensional vortex shedding. As was indicated by Smeed's results, rotation combined with stratification can help to enforce two-dimensionality even though Rossby numbers are not small. Accordingly, Etling classified the observations into Rossby, Ekman and Burger number regimes.

Little is known about geostrophic flow over an array of obstacles representing a rough bottom. Niino* studied this situation experimentally in a rotating circular tank with a differentially rotating lid. Over a wide range of $\Delta\Omega$ versus Ω he observed axisymmetric rolls whose structure was nearly depth independent. For larger rotation rates and larger values of differential rotation the rolls were found to become unstable with respect to waves with speed nearly equal to the mean flow. Niino also measured bottom stress for various spacing of the obstacles.

5. Vortex flow and centrifuges

Four papers were concerned with intense vortices and vortex breakdown, much studied in the context of aeronautical applications. A well-defined experiment in this context is the confined flow in a cylindrical container driven by a rotating disc at one endwall. Spohn, Mory & Hopfinger* presented results of the flow structure and of vortex breakdown conditions with and without a free surface, and for an aspect ratio of depth H to radius R of 2. Breakdown, characterized by an on-axis stagnation point followed by a recirculating region occurred at a Reynolds number $Re = \Omega R^2/\nu \geq 1440$ in the case of a rigid lid, and at $Re \geq 1350$ in the case of a free surface. In general the critical Reynolds number depends on aspect ratio (see Escudier 1984). With a rigid lid, breakdown conditions were shown to be consistent with Benjamin's (1962) finite transition theory. When the boundary is a free surface, Benjamin's theory is not applicable because the breakdown bubble is attached to the free surface and no supercritical flow regime seems to exist. The difference in breakdown with and without a free surface is illustrated in figure 1. As the Reynolds number is increased, the flow becomes unsteady. With a free surface unsteadiness sets in earlier (above a Reynolds number of about 2200).

Daube* presented numerical results of an axisymmetric calculation for the same flow with $H/R = 2$. With a rigid lid, onset of vortex breakdown is at a Reynolds number which is close to the experimental value. In the case of a free surface, however, the calculated critical Reynolds number is well below ($Re \approx 800$) the experimental value. Daube was particularly interested in the different bifurcations which the flow undergoes when the Reynolds number is increased. He showed that, with a rigid lid, transition to an oscillatory regime occurs at a critical Reynolds number of 2400 and that this transition has a supercritical Hopf bifurcation behaviour. The frequency of oscillation is f_1 for $Re \in [2400, 3500]$ and f_2 for $Re > 4250$. The change from one fundamental to the other has a hysteresis behaviour. Period-doubling phenomena were also observed (Daube & Sorensen 1989).

The paper by Pagan & Solignac* emphasized a vortex breakdown phenomenon in an aerodynamic context where the vortex was generated either by a delta wing or blades set at an angle of attack and subjected to an adverse pressure gradient. The blades were attached to a body of revolution through which air could be injected, allowing control of the axial velocity distribution independently of the circulation.

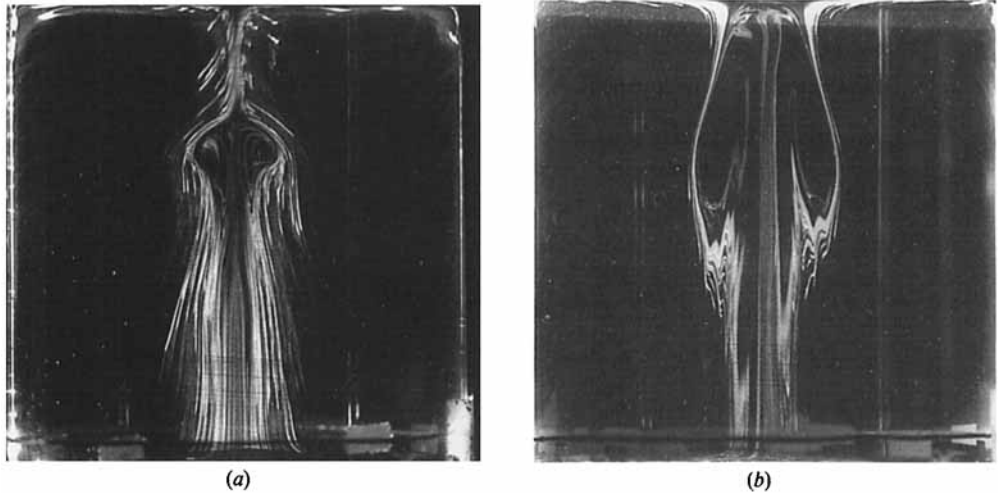


FIGURE 1. Effect of boundary conditions on vortex breakdown produced by a rotating disc in a cylindrical container: (a) with a rigid cover; (b) with a free surface. (Spohn *et al.**). Both photographs were taken at $Re = 1850$ and $H/R = 2$. The flow is visualized by the electrolytic precipitation technique.

Detailed measurements were obtained using a laser-Doppler anemometer, which showed the dependence of breakdown conditions on pressure gradient and on axial flow with respect to azimuthal velocity profiles. Numerical calculations in (ψ, ω) variables showed good agreement with experiments (Pagan & Benay 1988).

Although the problem treated by Avellan* was even further away from the main topic of the conference than the foregoing paper, it stimulated much interest on the part of the participants. It had been included in the conference because it helps to understand cavitation in rotating hydraulic machines. Avellan presented results of cavitation at the leading edge of a hydrofoil placed at an incidence angle with respect to the oncoming high-speed flow. Detachment of the stationary cavity at the leading edge occurs through a spanwise distribution of cavitating spots which develop downstream to form cavitating cones. These cone-shaped cavities interact with spanwise vortices situated at the edge of the stationary cavity, and deform these vortices into intense hairpin-shaped cavitating vortices which then break off the stationary cavity (Avellan, Dupont & Ryhming 1988). Visualizations also seem to indicate the existence of streamwise vortices which originate near the leading edge. During the discussion it was speculated that these vortices could be caused by Görtler instability due to the concave curvature in the mean flow just upstream of the leading edge. The cone-shaped spots seem to originate randomly in these vortices.

Very widespread engineering applications of rotating flows are encountered in the context of cyclone separators. Because of their importance there exist many guidelines for the design of efficient centrifuges but detailed investigations of the flow structure are not often found. There is room for fundamental studies and improvement of performance. One illustration of this situation was presented by Borgström*. The high-speed centrifuges constructed by 'Alfa-Laval' contain in the central part a stack of closely spaced conical discs which contain, for structural reasons, radial spacers called 'caulks'. The standard model used to calculate the flow structure predicted values for the pressure drop which were about twice the measured value. Borgström was able to show that the smaller pressure drop is due to the wake of the radial spacers. Without the spacers the flow is essentially confined

to thin Ekman layers on the discs. The vortices of the wake pump fluid into the central region hence reducing the pressure loss by increasing the effective flow section.

Theoretical work exploiting the Boycott effect in hydrocyclones has been initiated by Greenspan during the past few years (see, for instance, Greenspan & Ungarish 1985). The idea is to replace the gravity force by the centrifugal force and the effect of inclined container walls (the Boycott effect) by conically shaped cylinders. Because of the Coriolis force, conical walls do not enhance separation performances when the spacing between the walls is large and the Coriolis force is not counteracted by viscous shear. Another means to counteract the Coriolis force is by meridional barriers. Amberg* has studied this flow theoretically. He presented results for the boundary-layer flow on the conical wall and for the corner boundary layer at the intersection between the conical wall and the meridional barrier. Expressions for the thickness of the clear fluid layers were also obtained (see Amberg & Greenspan 1986). Experiments with two homogeneous fluids of different densities were in qualitative agreement with theory.

6. Convection in rotating fluids

Thermally driven flows with strong rotation effects are of fundamental importance in planetary circulations and also in stellar systems. Aspects of this were discussed during Euromech 106 (Hopfinger, Atten & Busse 1979). It is well known that the atmospheric circulation is driven by the temperature difference between the equatorial and polar regions and the resulting flow in mid-latitudes can be simulated by differential heating of a fluid annulus, demonstrated in a long series of experiments by R. Hide and his group and also by others. In his lecture, Hide* reviewed briefly some of these experiments and the different ramifications of this annulus experiment. On the theoretical side it led to detailed studies of baroclinic instabilities, dynamical systems and chaos, geostrophic turbulence and topographic effects, and on the applied side various configurations have been investigated and, in particular, the sensitivity of the circulation to changes in boundary conditions and topography. Hide focused his lecture on the effect of a radial barrier across the annulus gap. He showed that in the presence of a barrier the total heat transport across the gap is virtually independent on rotation rate Ω . As Ω increases from low to high values, the flow patterns range in complexity from spatially and temporally regular to highly chaotic, but the total heat transport is not altered. Without a barrier the heat flux decreases with increasing rotation rate. Hide described the topological characteristics of the flow by which it is possible to maintain a constant heat flux.

Zhang & Busse* reported on thermal convection in a rotating sphere, being interested in the transition to turbulence and the generation of a magnetic field. The basic state is spherically symmetric and, as the Rayleigh number is increased, different classes of solutions were identified: vacillation solution, mixed azimuthal symmetry, time symmetry.

Rieutord* obtained a class of solutions of linearized equations governing the flow in spherical geometry. Instead of using an expansion in Ekman number, Rieutord looked for solutions from an expansion in spherical harmonics. These solutions split naturally into interior solutions (almost inviscid) and boundary-layer solutions. In this way, problems of interest such as the spin-up flow in a sphere, modes of convection at marginal stability, inertial modes or linear MHD flows can be solved.

The next two papers were concerned with experiments on convection in a horizontal rotating fluid layer with the rotation axis being parallel to gravity. Boubnov & Golitsyn* conducted detailed measurements of and flow visualization in such a system for Rayleigh flux numbers Ra_f of 10^6 to 10^{12} and different Taylor numbers, Ta , in continuation of their earlier work (Boubnov & Golitsyn 1986). The different flow regimes were classified in a Ra_f - Ta diagram. In the high Ra_f , high Ta regime (geostrophic turbulence regime), corresponding to the irregular vortex regime, the mean temperature was found to be independent of height in the fluid interior as in non-rotating convection. The temperature frequency spectra had the same shape as in the non-rotating regime, but shifted to lower frequencies with increasing rotation rate. The spectra showed no power-law inertial range. In the regime with regular vortex structure, the temperature decreased linearly with height and the frequency spectra exhibited at high frequencies, ω , an ω^{-2} behaviour (Boubnov & Golitsyn 1988). The flow visualization showed qualitatively the structure of convection and the localization of the ascending and descending fluid regions.

Fernando, Chen & Boyer* investigated the same problem as Boubnov & Golitsyn but with the Rayleigh flux number being two to three orders of magnitude higher ($Ra_f = 10^{12}$ to 10^{14} with $Ta = 10^8$ to 10^{11}). At these Rayleigh and Taylor numbers the temperature frequency spectra show a $-\frac{5}{3}$ inertial range over about one decade. Rotation decreases the turbulence intensity in a layer above the transition height $h_c = 4.5(q_0/\Omega^3)^{\frac{1}{2}}$ where q_0 is the imposed heat flux, and turbulence data collapsed well when the length and velocity scales $(q_0/\Omega^3)^{\frac{1}{2}}$ and $(q_0/\Omega)^{\frac{1}{2}}$, respectively, were used for normalization. Visualizations showed the existence of intense vortices similar to those observed by Hopfinger *et al.* (1982).

Numerical calculations of convection in a differentially heated, rotating annulus were performed by Le Quéré & Pécheux*. A three-dimensional algorithm was used with Tau-Chebyshev polynomials in the meridional plane and Fourier decompositions in the azimuthal direction. The time-dependent Boussinesq equations were integrated for an aspect ratio of 1 and Rayleigh numbers up to 10^6 with Taylor numbers of order 10^6 . The baroclinic instability modes 3-6 were calculated and the flow became unsteady at $Ra \approx 10^6$ (see LeQuéré & Pécheux 1989).

Randriamampiana, Chaouche, Segura & Boutoux* also calculated the flow in a differentially heated, rotating annular cavity but with the primary motivation of understanding and improving the cooling system of gas turbine rotors. Closed cavities, cavities with throughflow and cavities with differential rotation and through-flow were studied. The numerical techniques used were a high-resolution, axisymmetric spectral code in the case of the closed cavity and a finite-element code in the case of the open cavities. It was also desirable to resolve the flow in the very thin Ekman layers and up to the transition to time-dependent flow regimes. Results were presented of the laminar flow structure and the heat transfer in the different cases considered.

Convection from an isolated source in a rotating fluid was described by Muller*. Contrary to previous studies of thermals or plumes in a rotating fluid, released at the axis of rotation, here the buoyant source was off-centre, the axis of rotation being parallel to gravity. The buoyant fluid is thus subjected to the axial gravity force and a lateral centrifugal force. To some extent this simulates tropical cyclones. Both, short (thermals) and long (plumes) duration heat sources were studied. The effect of rotation, other than the inward deflection, is to make the thermal or plume more compact (Muller & Burch 1985). Muller also reported oscillations of frequency

proportional to Ω^2 which he attributed to inertial waves generated by the buoyant source which then interact with the plume.

7. Geostrophic vortices

The study of geostrophic vortices has advanced significantly in recent years, particularly inspired by observations of mesoscale oceanic eddies. Satellite infra-red images of the sea surface have revealed a wealth of eddy structures and have raised questions concerning the formation, propagation and ultimate destruction of vortex structures. Much of the work on this topic has involved laboratory simulations and these were reviewed by van Heijst*.

The production of vortices by baroclinic instabilities was briefly mentioned, but attention was mainly restricted to the case of isolated vortices in homogeneous fluids. Two main methods of generation were identified: viz, the use of sources and sinks to produce anticyclones and cyclones, respectively, and vortices produced by stirring a localized region. In each case the main core of the vortex is surrounded by a ring of fluid containing the opposite vorticity. The structure of the vortices both in the laboratory and in numerical models (e.g. McWilliams 1984) exhibits a nonlinear relationship between the vorticity and the stream function.

The stability of these vortices was described. Anticyclones are observed to split into two dipoles in agreement with numerical work by Flierl (1988). Cyclonic vortices exhibit more variety, with a transition to a tripolar structure occurring when the ring of negative vorticity is narrow.

The paper concluded with a discussion of the effects of bottom topography. A uniform slope, which simulates a β -plane, produces a NW drift of cyclones and a SW drift of anticyclones. For cyclones generated in a tank with a parabolic free surface, this effect causes a drift towards the centre of the tank. It was suggested that this phenomenon may explain the ubiquitous merging of cyclonic vortices, observed by Griffiths & Hopfinger (1987).

The papers following this review began with a presentation of a mathematical approach to the structure of confined two-dimensional vortices by van Groesen*. Exact solutions of the Euler equations were sought that extremize the kinetic energy. Maxima in the kinetic energy as friction decreases the angular momentum are possible only for cyclones, and families of bi-polar, tri-polar and four-polar structures are possible. Anticyclones do not possess this property, suggesting that they always break up, as seems to be the case observationally.

Dritschel* described the effects of a large-scale shear flow on a two-dimensional vortex, when the vorticity in the shear and the vortex have opposite signs. The classical result of Rayleigh (1894) is that vortices are linearly stable when the imposed shear is sufficiently strong. Dritschel showed that the result holds for nonlinear flows, and suggested that this stabilization explains why thin vortex strips wrapped around vortices do not roll up as they would if isolated. He further suggested that vortices have sharp edges at high Reynolds numbers and that they are not as circular as numerical calculations suggest.

The merging of two baroclinic vortices was investigated numerically by Verron & Hopfinger*. Laboratory experiments (Griffiths & Hopfinger 1987) showed that two-layer baroclinic vortices of like signs merge when the distance between the vortices falls below a critical distance whose value depends strongly on the Rossby deformation radius. The experiments show that merging occurs at separation distances considerably larger than the critical distance for barotropic vortices when

the deformation radius λ is large compared to the vortex core size R . When these two scales are comparable the merging distance falls below the barotropic value and both these features are captured in the quasi-geostrophic model. The model also shows that the barotropic value is again recovered for $\lambda/R > 4$. An interesting property of the numerical model is that the conditions for merging are sensitive to the vorticity distribution in the two layers.

The final three papers in this session were directed towards an explanation of long-lived coherent vortices observed in planetary atmospheres, such as Jupiter's Great Red Spot. Sommeria, Meyers & Swinney* described a laboratory experiment in which a barotropic shear flow is produced on a topographic β -plane by an arrangement of sources and sinks in the bottom of an annular tank. A feature of this experiment is that the rotation rate is rapid so that frictional effects are very weak. Westward jets are very turbulent, and a wide region of cyclonic shear with uniform potential vorticity is produced. A large, persistent isolated cyclonic vortex emerges, and new cyclonic vortices which are produced merge with the main vortex (Sommeria, Meyers & Swinney 1988). The edge of the vortex acts as a barrier to the turbulent motion in the jet. Eastward jets focus into a narrow, wavy boundary separating two regions of quasi-uniform potential vorticity. The gradient of potential vorticity acts as a barrier to turbulent mixing, and suggests that there is competition between mixing and Rossby waves, akin to the effects of internal waves in stratified flows. Similar 'potential vorticity barriers' are believed to occur at the boundary of the polar vortex in the wintertime stratosphere.

An alternative mechanism to account for the existence of long-lived eddies was proposed by Read*. Extending earlier work of Read & Hide (1983) on thermal convection in a rotating fluid with internal heating, he presented new results which included, for the first time, the β -effect. Baroclinic instability led to the formation of coherent eddies which propagated relative to the mean flow, and with a scale significantly smaller than the scale of the apparatus. The eddies formed as a result of instabilities associated with streamline curvature and were observed to have significant baroclinic structures. A related two-layer quasi-geostrophic numerical model was presented by Lewis*. Many of the features observed by Read & Hide (1983) and Read* were reproduced in the model, with baroclinic eddies being produced by instabilities of the zonal flow. Ageostrophic effects, although small, cause the eddies to steepen and be maintained against dissipation. These results were also found in a deep atmospheric model. It was found that the Rhines' scale, at which energy cascade to large scale is blocked, implies competition between turbulent mixing and Rossby wave propagation in the manner suggested by Sommeria *et al**

8. Geostrophic adjustment and waves

The theme of this session was the response of rotating fluid to various types of forcing processes. The first three processes dealt with aspects of the spin-up problem. Maxworthy & Monismith*, motivated by questions raised in the management of reservoirs, described laboratory experiments on the selective withdrawal from a rotating, stratified (constant buoyancy frequency N , Coriolis parameter f) fluid. The transient response initiates Kelvin waves which propagate cyclonically around the perimeter of the reservoir. An anticyclonic withdrawal layer is established which grows in thickness with time, and they attributed this growth to the effects of vertical diffusion. In a stratified fluid, Ekman spin-up effects are limited to regions near the top and bottom boundary (Benton & Clark 1974). For a typical reservoir

$N/f \approx 10^2$, and so the spin-up process is slow, taking about one month. Interesting instabilities of the withdrawal layer were also reported, in which the flow was observed to break up into a series of counter-rotating gyres (Monismith & Maxworthy 1989).

Two extreme cases of the spin-up of a stratified fluid in a rotating cylinder were presented by O'Donnell & Linden*. The effects of a free surface on homogeneous spin-up were discussed first. Experiments were conducted over a wide range of rotational Froude numbers F , up to the limit ($F = 16$) when the free surface intersects the tank bottom on the axis of the cylinder. They found that the main characteristics of spin-up at low Froude number are retained, i.e. the relative azimuthal velocity decays exponentially with time and remains proportional to the radius. At high rotation rates, spin-up is delayed and the spin-up timescale increases linearly with F . The second case discussed was that of spin-up of two immiscible layers, and comparisons with the theory of Pedlosky (1967) showed good agreement. At sufficiently high Rossby numbers, transient baroclinic waves were observed on the interface and these waves were observed to accelerate the spin-up process.

Spin-up from rest, in which the initial motion can be described by potential flow, allows the effects of non-axisymmetric geometry to be calculated. van Heijst & Davies* derived the flow generated by radial boundaries in an annulus, and compared these calculated flows with their own laboratory observations. The agreement in the initial stages was good, but the experiments showed that flow separation occurred, leading to intense three-dimensional turbulent flow. As this motion decayed a transition to large-scale, two-dimensional eddy structures was produced by the background rotation. These separation phenomena are reminiscent of those reported by Maxworthy & Monismith*, although the connection between the two cases is unclear. The final spin-up was accomplished by Ekman layers at the base of these eddies.

The next two papers dealt with aspects of the adjustment process. Johnson, Hermann & Rhines* described the adjustment of a discontinuity in free surface height in a rotating channel. Linear theory (Gill 1977) predicts that a Kelvin wave travels down the right-hand side of the channel ($f > 0$), producing a gravity current, one Rossby deformation radius in width, along that sidewall. Using contour dynamics, Johnson *et al.** show that at finite amplitude, flow also occurs down the left-hand wall, and eventually fluid travels downstream everywhere across even wide channels. The basic reason for propagation on the left-hand side is as follows. During the initial adjustment, at finite amplitude, fluid travels a small distance downstream before being turned by the Coriolis force. Fluid columns beyond the line of the initial discontinuity are compressed, generating anticyclonic vorticity. An image cyclone is generated near the left-hand wall by the normal flow boundary condition, and the cyclone-anticyclone pair propagates downstream.

The steady exchange flow in a channel between two reservoirs containing fluid of different densities was discussed by Dalziel*. He described a theoretical and experimental study of the effect of background rotation on the classical hydraulic control problem. Rotation causes the interface to slope across the channel, and for wide channels can lead it to separate from one or both of the sidewalls. It is impossible to use the usual definition of Froude number F based on layer depths, and Dalziel* introduced an alternative definition, $F^2 = 1 + \gamma c_1 c_2$, where $\gamma > 0$ depends on the geometry and c_1, c_2 are the phase speeds of long waves on the interface. Supercritical flow ($F > 1$) requires that both c_1 and c_2 have the same sign and so information can only travel in one direction. In subcritical flows ($F < 1$) information

can travel in both directions along the interface. For channels narrow compared to the Rossby deformation radius the effects of rotation are small. For wide channels, on the other hand, separation occurs and the exchange flow between the two reservoirs is limited by the rotation and not by the width of the channel. In addition, the flow crosses over from one side of the channel to the other and the classical hydraulic approach does not apply in this limit. Applications of these ideas were applied to flow through the Strait of Gibraltar, where the effect of the Earth's rotation causes about a 10% reduction in the exchange flow between the Mediterranean and the Atlantic.

The oceanographic theme was continued in the next two papers. Darby & Willmott* discussed the effects of variable upper mixed-layer depth on the propagation of Rossby waves. A numerical model, in which the mixed-layer depth profile was varied according to data appropriate to the North Atlantic was described, and this was supported by asymptotic solutions obtained using ray theory. They found that the Rossby waves were highly distorted and that wave energy was concentrated in regions where the mixed-layer depth was greatest. Calculations were also reported on the effect of the geostrophic flow generated by the mixed-layer depth variations. McClimans* described the effects of flow beneath a surface coastal current. The current, a model of the Norwegian Coastal Current which is driven by a buoyant output from the Baltic Sea, is unstable to baroclinic frontal instabilities (Griffiths & Linden 1981), and the effects of these instabilities have important economic consequences for oil drilling operations in the area. McClimans discussed laboratory simulations which indicated that the instabilities were inhibited by the underflow, and perhaps also by the bottom topography.

The final two papers in this session discussed the effects of inertial waves generated in closed rotating containers. Aldridge*, motivated by a wish to interpret gravimetric observations of 13–16 hour-period oscillations thought to be associated with oscillations of the Earth's liquid core, discussed mechanisms for generating inertial waves of these periods. He suggested that wobble modes of the Earth generated inertial modes, and described a model which includes nonlinear interactions between modes and viscous effects. Identification of low-order modes with the gravimetric data looks promising.

A detailed experimental investigation of the nonlinear phenomena associated with small-amplitude forcing of a rotating cylinder was described by Manasseh*. The application of this work is to possible instabilities of satellites due to fluid motions in fuel tanks. A right circular cylinder undergoes forced precession and the 'resonant collapse' regimes (McEwan 1970) were examined by flow visualization. A rich variety of collapse regimes were identified, depending primarily on the frequency of the precessional forcing. Two examples are shown in figure 2. A dramatic video of the flow was shown illustrating recurrent collapse where the flow becomes reordered after each collapse, which engendered much discussion.

9. Baroclinic and barotropic instabilities

The final session of the Colloquium was concerned with instabilities of rotating flows. This topic had been mentioned in a number of earlier papers, but in the present group of papers the instability was the prime concern of the work. A review of baroclinic instability was scheduled but had to be cancelled at the last moment. A written version of it will appear (Klein 1990). This will be a welcome contribution since reviews of this kind seem to be lacking.

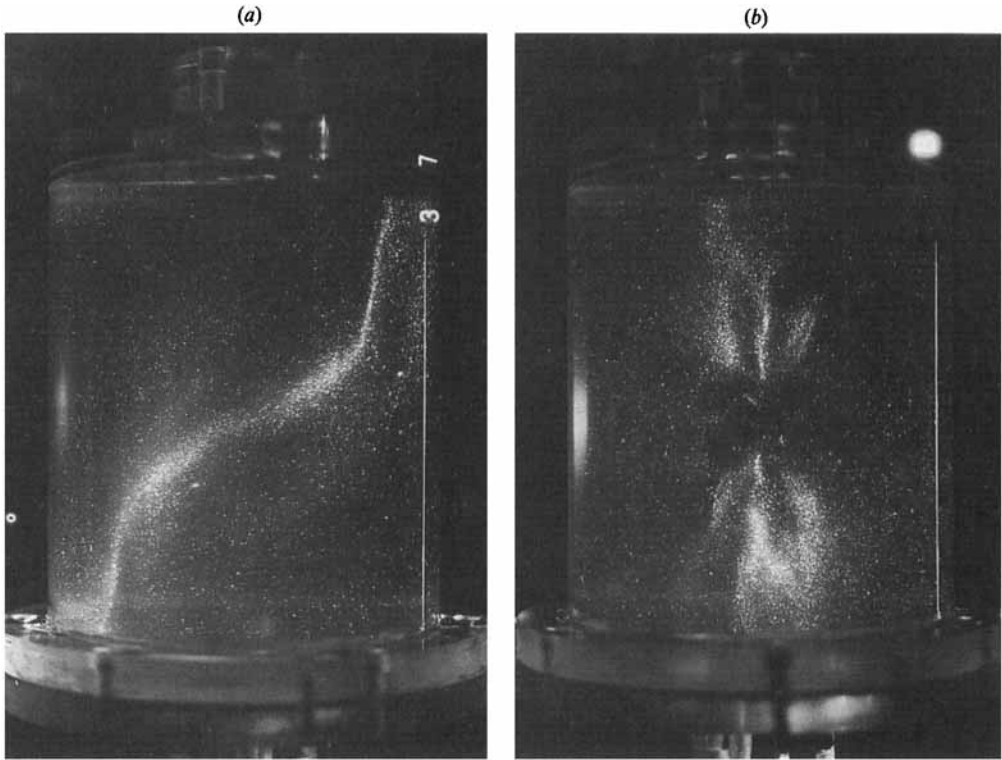


FIGURE 2. Resonant collapse in a precessing cylinder: (a) collapse of the $(1, 1, 1)$ mode at 4.5 revs. after forcing; (b) collapse of the $(3, 1, 1)$ mode, 13 revs. after forcing.

Lorenzen, Meier & Assenheimer* described their recent work on the classical heated annulus flow. Velocity measurements were made with an LDV system and thermochromic liquid crystals were used to obtain a qualitative picture of the temperature field. Experiments with a free surface were contrasted with those in which the upper boundary was a rigid lid. Similar behaviour was observed in the two cases, although the zonal wavenumber was generally greater with the rigid lid. Wavenumber hysteresis was observed with changes in rotation rate.

Further work on baroclinic instability was presented by Brindley, Grimson & Moroz* who examined the nonlinear stability of a three-layer channel flow. Near the linear stability, they found an interaction between two waves with the same wavenumber but different phase speeds which gave rise to amplitude vacillation and nonlinear equilibration. This process seems to capture the essential features of amplitude vacillation observed in rotating annulus experiments (see e.g. Buzyna, Pfeffer & Kung 1984). They noted that the approach to equilibrated form may either be monotonic or in the form of a limit cycle, and that the vacillated state is height dependent.

The final three papers examined the stability of vortex structures. Carton* discussed the baroclinic and barotropic instability of quasi-geostrophic circular vortices. He found, using a spectral model, that small initial perturbations lead to amplitude vacillations, while large initial disturbances lead to breakup into dipoles and tripoles as observed experimentally by Griffiths & Linden (1981), van Heijst*

and others. Intermediate behaviour is found between these two extremes, with stabilization occurring at finite amplitude.

The instability within a elliptical vortex was discussed by Craik*. The classical result is due to Kelvin (1887) who showed that flows with constant vorticity and elliptical streamlines are unstable to a planar disturbance in which the wavenumber varies in time. In a fully nonlinear analysis, the effect of Coriolis forces on this instability were examined. Craik showed that when the background vorticity is of the opposite sign to that in the vortex the instability is enhanced. The instability corresponds to bands of inertia waves which propagate around the ellipse. When the background vorticity is of the same sign as that in the vortex, the instability is inhibited, and can be totally suppressed for certain ranges of rotation rates (Craik 1989)

This theme of the instability of elliptical vortices was continued in the final paper of the meeting by Gledzer, Dolzhanskey & Obukhov*. They showed striking photographs of the breakdown of flow in a elliptical container which is brought to rest from a state of solid-body rotation. This instability is a long-wavelength one in contrast to the instability discussed by Craik which is at wavelengths short compared with the vortex diameter.

10. Conclusion

Comparing this meeting with the IUTAM meeting held 25 years ago (Bretherton *et al.* 1966), it is striking that a number of topics discussed at that meeting continue to be actively researched. Indeed some of the participants at the IUTAM meeting contributed to this Colloquium, and their fascination for the subject remains undiminished! This is particularly the case for rotating annulus experiments which, while the emphasis has changed as a result of recent advances in the theory of dynamical systems, have remained virtually unchanged. Perhaps the most striking advances have occurred in the understanding of coherent structures (vortices) in rotating flows. Much of the motivation for this work has come from geophysical observations, particularly in the oceans. Complex, long-lived structures such as dipoles and tripoles have been discovered and their importance in the transporting of angular momentum and mass have been recognised. Numerous generation mechanisms have been identified, and the dynamics of these nonlinear structures and their interplay with linear processes, such as Rossby waves, is beginning to be understood. Clearly, these structures play a large role in environmental and climatic processes and they will continue to be a focus of future work. Many questions remain, particularly their longevity in the face of numerous decay processes.

Another area of significant progress is that of turbulence modelling in rotating flows. Both direct and large-eddy simulations are now possible, and comparisons have been made with stratified turbulence and curved flows. In particular, the anisotropization with lengthscales parallel to the rotation vector increasing, have been modelled and compared with results from wind-tunnel experiments. The effects of rotation appear to be well accounted for by considering the local turbulence Rossby number, analogous to the Froude number for the collapse of stratified turbulence. The production of strong, isolated vortices has been identified, and these bear striking resemblances to structures observed in curved flows. The role of inertial waves in energy transfer has been identified in isolated circumstances, but a general result is not yet available.

Progress has also been made in investigations of barotropic and baroclinic instabilities. The stability of coherent structures is now receiving considerable attention, and these studies will undoubtedly answer some of the questions concerning the decay of these structures. There now appears to be a clear connection between the formation of structures and instabilities of basic flows, but the nonlinear dynamics is still unclear in many circumstances. Vortex breakdown illustrates this point nicely, being a relatively well-controlled phenomenon exhibiting a wide range of complex nonlinear phenomena.

A striking impression of the Colloquium was provided by the many beautiful visualizations of the flows. These were both experimental and computational results, which revealed a wealth of unexpected and fascinating phenomena. Recent advances in computational power have produced calculations of flows in complex situations which allow us to investigate the full diagnostics of the flow fields. Similarly, advances in experimental techniques, particularly in the area of image processing, are producing laboratory measurements of similar quality. This interplay between theory and experiments and our new understanding of nonlinear dynamics are likely to lead to new insights on these important problems. In a subject which is still in its infancy, and in which, as this Colloquium showed, new and unexpected phenomena are continually being discovered, we expect further significant advances to be made. We trust that we shall not have to wait a further 25 years for the next meeting on this topic.

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